TeVeS/MOND is in harmony with gravitational redshifts in galaxy clusters

Bekenstein, Jacob D.; Sanders, Robert H.

Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
10.1111/j.1745-3933.2011.01206.x

Publication date:
2012

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY LETTERS

Mon. Not. R. Astron. Soc. **421,** L59–L61 (2012) doi:10.1111/j.1745-3933.2011.01206.x

T*e*V*e*S/MOND is in harmony with gravitational redshifts in galaxy clusters

Jacob D. Bekenstein[1] and Robert H. Sanders[2]⋆

[1]*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*
[2]*Kapteyn Astronomical Institute, Groningen, the Netherlands*

Accepted 2011 December 9. Received 2011 November 30; in original form 2011 November 3

ABSTRACT

Wojtak, Hansen & Hjorth have recently claimed to confirm general relativity (GR) and to rule out the tensor–vector–scalar (T*e*V*e*S) gravitational theory based on an analysis of the gravitational redshifts of galaxies in 7800 clusters. However, their ubiquitous modelling of the sources of cluster gravitational fields with Navarro–Frenk–White mass profiles is neither empirically justified out to the necessary radii in clusters, nor germane in the case of T*e*V*e*S. Using MONDian (where MOND stands for MOdified Newtonian Dynamics) isothermal sphere models consistently constructed within MOND (equivalent to T*e*V*e*S models), we can fit the determined redshifts no worse than does GR with dark haloes. Moreover, Wojtak, Hansen & Hjorth have inappropriately used the simple MOND interpolating function for the μ function of the scalar field of T*e*V*e*S; the consequent MOND effective interpolating function turns out to enhance the gravitational potential, and so contributes to the discrepancy which forms the basis of their claims.

Key words: gravitation – galaxies: clusters: general – dark matter.

In a recent paper, Wojtak, Hansen & Hjorth (2011, hereafter WHH) analyse the pattern of spectroscopic redshifts for galaxies in 7800 clusters from the Seventh Data Release of the Sloan Digital Sky Survey. The motion of a galaxy in a cluster engenders a redshift or blueshift of its spectral lines; the equivalent velocities are typically of the order of 600 km s^{-1}. WHH make a good case that they are able to dig out the superimposed gravitational redshift, which is equivalent to a velocity of the order 10 km s^{-1}, by stacking data from all clusters and measuring the displacement of the redshift distribution's centroid with growing radial coordinate r in the typical cluster.

However useful their technique may turn out to be, WHH draw from it erroneous conclusions that: (i) general relativity (GR) is confirmed by the observed redshift pattern; (ii) the $f(R)$ gravity theory (Carroll et al. 2004) may well be compatible with the data; and (iii) the tensor–vector–scalar theory (T*e*V*e*S, Bekenstein 2004; Skordis 2009), an early relativistic gravity implementation of the MOdified Newtonian Dynamics (MOND) paradigm (Milgrom 1983), which claims to describe galaxy dynamics without dark matter, is ruled out. In the discussion that follows, we will refer to MOND and Newtonian masses, meaning the dynamical mass required by each named theory to account for the kinematics of galaxies or gas in a cluster.

Let us recall that in the above-mentioned theories, as well as in any other metric gravitational theory endowed with the usual weak field limit, $g_{00} \approx -c^2 - 2\Phi$, the gravitational redshift is quantified, to first order, by the same gravitational potential Φ which controls the motion of galaxies in their parent clusters, etc. (Misner, Thorne & Wheeler 1971). The gravitational redshift suffered by a galaxy situated at radial position r should follow from the formula $z = -\Phi(r)c^{-2}$ [with $\Phi(\infty) = 0$; gravitational potential is negative]. This is a straight consequence of the Einstein equivalence principle which is implemented in all such theories (Will 1986). We note that the gravitational redshift combined with gravitational lensing could be a test of theories in which the two Newtonian potentials (the first-order terms of g_{00} and g_{rr}) differ. This is not the case with T*e*V*e*S or with other suggested relativistic extensions of MOND, such as Einstein–Aether theories or BIMOND, and, in any case, it is not a comparison done by WHH.

WHH determine $\Phi(r)$ in a cluster, by matching the dispersion of galaxy velocities in their stacked cluster with the best-fitting NFW halo model (which emerges from cosmic N-body simulations within GR, Navarro, Frenk & White 1997), and appealing to a standard assumption about the predominant shapes of galaxy orbits. Actually they determine the gravitational shift relative to that of light emerging from the centre of the cluster, so their result is presented as a blueshift: $\Delta z = [\Phi(0) - \Phi(r)]/c^2 < 0$. Now, were $\Phi(r)$ inferred directly from galaxy velocity dispersions, the curve $\Phi(r)$ versus r would have to be the same for all metric theories with a standard weak field limit. Thus, how did WHH get their fig. 2 where the redshift curve for T*e*V*e*S lies well below GR's?

While the gravitational redshift by itself is in no sense a test of competing theories of gravity (as long as all respect the equivalence

⋆E-mail: sanders@astro.rug.nl

principle), it can be a test of the mass distribution in the outer parts of clusters, that is, the mass distribution that may be implied by a particular theory. For clusters in general, the mass distribution can be measured dynamically, by the motion of galaxies or the density and temperature distribution of hot gas, but these techniques work out to $r = 1$–2 Mpc at most. Yet WHH purport to observationally determine the gravitational redshift, and hence the potential, out to 6 Mpc. What do they assume about the mass distributions in these outer regions in light of competing theories of gravity?

For GR with dark matter, WHH *assume* that the mass distribution out to 6 Mpc, four times the typical virial radius, is well described by the popular NFW profile (Navarro et al. 1997). They pick that NFW model, specified by a virial radius and a concentration parameter, which best fits the velocity dispersion data of the average cluster within about 1 Mpc from the centre. Now in a NFW halo beyond the virial radius of 1–2 Mpc, the density mass distribution falls as $1/r^3$. It must be emphasized that this behaviour is an assumption; there is no independent tracer of the mass distribution out to 6 Mpc where the cluster may not be in virial equilibrium and may still be accreting.

MOND/T*e*V*e*S is less flexible; as presently understood, the theory unavoidably yields an asymptotically logarithmically rising potential which is equivalent to a 'phantom' dark matter density falling as $1/r^2$ in the outer regions (phantom in the sense of dark matter that one would suppose to be present were one to analyse the problem Newtonianly). Thus, the *effective* density declines more slowly than for the assumed NFW profile. One might suppose that this is the reason for the different curves in fig. 2 of WHH: MOND/T*e*V*e*S produces more potential in the outer regions of clusters than does GR for an equal true mass distribution in the inner regions. In their supplementary material, WHH admit that the mass distribution is uncertain in the outer regions but claim that the GR result is fairly insensitive to the actual exponent of a power-law density distribution.

However, the actual problem is primarily one of scaling. In their test of the MOND/T*e*V*e*S scenario, WHH again assume a NFW mass distribution, but one scaled down to 80 per cent of the Newtonian dynamical mass. That is to say, the discrepancy between the observed and dynamical masses is reduced from a factor of 6 to a factor of 5 (in clusters some unseen matter is also required by MOND, Sanders 2003). This assumption is based on an analysis of clusters by Pointecouteau & Silk (2005). However, as pointed out by Milgrom (2008), these authors used too small a value of a_0. The actual ratio of MOND to Newtonian mass should be more like 50 per cent (see fig. 10 of Sanders & McGaugh 2002, for results based upon about 90 X-ray-emitting clusters), and this factor decreases with radius (Angus et al. 2007).

However, the NFW profile is not particularly germane to the MOND/T*e*V*e*S world view; the profile arises in GR cosmological simulations. Therefore, we have calculated Δz using instead as source the mass distribution of a MONDian isothermal sphere with a velocity dispersion of 600 km s^{-1}. The MOND potential difference was calculated using the MOND equation (Milgrom 1983, here and henceforth $\Phi' \equiv \partial\Phi/\partial r$ and $\Phi_{\rm N}$ is the usual Newtonian potential)

$$\tilde{\mu}(\Phi')\,\Phi' = \Phi_{\rm N}', \tag{1}$$

with the 'standard' interpolating function (Milgrom 1983; Famaey & Binney 2005)

$$\tilde{\mu}(\Phi') = \frac{\Phi'/a_0}{\sqrt{1+\Phi'^2/a_0^2}}. \tag{2}$$

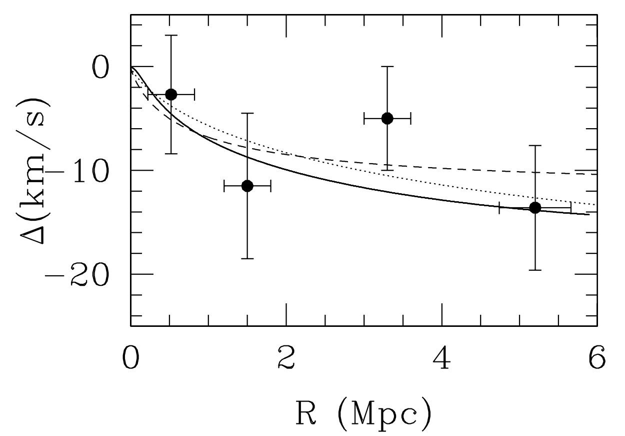

Figure 1. Gravitational redshift with respect to the centre of the cluster following WHH. The dashed curve is that of GR plus the NFW model which best fits the run of velocity dispersion in the central regions of the average cluster. The solid curve is that of the MOND/T*e*V*e*S isothermal sphere with a central velocity dispersion of 600 km s^{-1}. For comparison, the dotted curve shows the NFW model with mass reduced by 36 per cent in the context of T*e*V*e*S/MOND. The data points are from WHH's paper.

The total mass of our MONDian isothermal sphere is 1.45×10^{14} M$_\odot$ (MONDian isothermal isothermal spheres have finite mass). By comparison, we require a mass of about 4×10^{14} M$_\odot$ for the NFW halo out to 6 Mpc to emulate WHH's GR results. Thus, the MOND dynamical mass is only 36 per cent of the Newtonian dynamical mass within 6 Mpc, not 80 per cent.

The results are displayed in Fig. 1 along with the data of WHH (comparable to their fig. 2). The line-of-sight integration for the mean gravitational redshift at a particular projected radius was, following WHH, carried out on the assumption that the galaxies were distributed as the total mass distribution. The solid curve is the MOND prediction and the dashed line is that of GR plus NFW haloes. We see that, while the MOND curve does lie below the GR curve, both are consistent with the observations. The data do not distinguish between the predictions of the two theories because the natural mass distribution in the outer regions is different for GR and MOND. The two curves almost coincide within 1 Mpc from the centre, as they should, since the mass distributions and potentials coincide here, becoming different only in the outer regions.

Independent of the issue of the mass distribution in the outer reaches of a cluster, WHH made a conceptual error in their specifically T*e*V*e*S calculation of the gravitational redshift. In GR, Φ coincides with $\Phi_{\rm N}$, the latter being sourced by baryonic and dark matter. Although T*e*V*e*S comprises two metrics, primitive and physical, light and matter propagate exclusively on the physical metric. The potential Φ is related to the physical metric in just the way $\Phi_{\rm N}$ is related to GR's metric. T*e*V*e*S also includes a scalar field ϕ satisfying a non-linear field equation, formulated on the primitive metric, which is defined with the help of a specific function μ of the gradient of ϕ. This equation has the consequence that in a spherical cluster

$$\mu(\phi')\,\phi' = \frac{k}{4\pi}\,\Phi_{\rm N}', \tag{3}$$

where k is a small coupling constant (Bekenstein 2004) which WHH assume to be $k = 0.01$. As in the MOND paradigm, $\Phi_{\rm N}$ here is considered to be sourced only by the baryonic matter. The two metrics in T*e*V*e*S are so related that to linear order

$$\Phi = \Phi_{\rm N} + \phi. \tag{4}$$

By contrast to WHH's procedure to be summarized below, in our T*e*V*e*S analysis of the gravitational redshift, we have worked directly with the full MOND equation, equation (1), using the 'standard' form of $\tilde{\mu}$. This procedure is entirely permissible when spherical symmetry is obtained since, as shown by equation (4), the scalar ϕ plus the Newtonian potential combine in T*e*V*e*S to provide the effective weak field potential Φ, which yields directly the gravitational redshift. It is unnecessary to consider the scalar field ϕ separately as WHH did.

While WHH took Φ_N in their T*e*V*e*S analysis to be a fraction 80 per cent of the Φ_N required by the velocity dispersions in Newtonian gravity, we have argued above that the fraction should be more like a third. To determine Φ from equation (4), WHH computed ϕ through equation (3) using the prescription

$$\mu(\phi') = \frac{\sqrt{k\, l \phi'}}{1+\sqrt{k\, l\phi'}}, \tag{5}$$

where l is the T*e*V*e*S scale of length [whose relation to the MOND acceleration scale WHH take to be $a_0 = \sqrt{\kappa}/(4\pi l)$ based on Bekenstein 2004]. They justify this choice of μ because in MOND's versatile equation (1) an often-used choice is the 'simple' interpolating function (Famaey & Binney 2005)

$$\tilde{\mu}(\Phi') = \frac{\Phi'/a_0}{1+\Phi'/a_0}. \tag{6}$$

However, there is a confusion here. In T*e*V*e*S μ and $\tilde{\mu}$ are different functions (Bekenstein 2004). WHH's choice (5) corresponds, by equations (59)–(61) of that reference, to the relation

$$\Phi'/a_0 = \frac{\tilde{\mu}}{1-\tilde{\mu}}\;\frac{1}{1-\left(1+\frac{k}{4\pi}\right)\tilde{\mu}}. \tag{7}$$

This being quadratic, WHH's choice of $\mu(\phi')$ corresponds to a *double-valued* interpolation function $\tilde{\mu}(\Phi')$. One of its branches is physically ruled out because it gives $\tilde{\mu} > 1$. The permissible branch is plotted in Fig. 2 for WHH's choice $k = 0.01$. For a given source, it implies a significantly stronger gravitational field than do either of the well-tested 'simple' and 'standard' functions also shown (see equation 1). It is thus no wonder that WHH predict too deep a cluster potential, as reflected by their T*e*V*e*S gravitational redshift curve in fig. 2 of WHH.

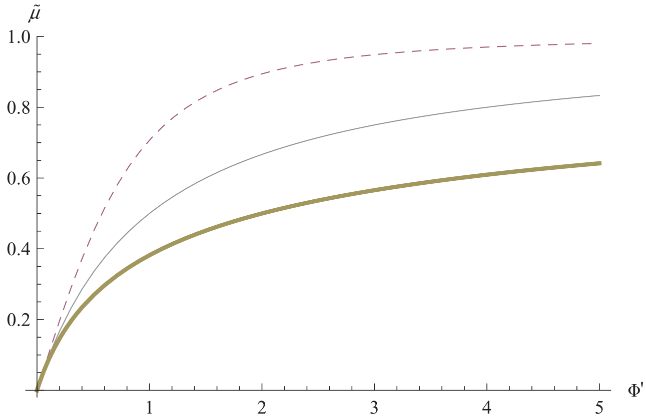

Figure 2. In this plot of $\tilde{\mu}$ versus Φ' (in units of a_0), the thick solid curve depicts the interpolation function for WHH's choice of $\mu(\phi')$ (equation 5) and $k = 0.01$, while the light solid and dashed curves are for the well-tested 'simple' and 'standard' interpolation functions, equations (6) and (2), respectively.

However, how does WHH's implied MOND interpolation function, when combined with a mass distribution natural to MOND, fare in light of the cluster data? We have repeated our calculation using a MONDian isothermal sphere source with MOND's equation (1), but this time with the interpolation function represented by the solid thick curve in Fig. 2. A source with 80 per cent of the Newtonian mass does lead to agreement with WHH's gravitational redshift *prediction* of $-22\,\mathrm{km\,s^{-1}}$ at the cluster centre, but pushes the predicted central galaxy velocity dispersion up to $850\,\mathrm{km\,s^{-1}}$, a number excluded by the data. The dispersion can be brought down to the observed level by reducing the source mass to 20 per cent of Newtonian, in which case the central gravitational redshift is down to $-13.6\,\mathrm{km\,s^{-1}}$ and the redshift curve is a bit closer to the GR one than shown in Fig. 1. Use of WHH's implied interpolation function undeniably eases MOND's need for dark matter in clusters. It should be remarked, however, that WHH's interpolation function has hardly been tested with galaxy rotation curves, an area in which MOND with the usual interpolation functions is very successful.

To conclude, WHH's far-reaching claims do not stand up under close scrutiny. The NFW profile which they assume everywhere is not supported by independent evidence out to the radial distances which they explore, and its use in the context of T*e*V*e*S is unnatural. In our test of T*e*V*e*S/MOND, a mass distribution consistently constructed within MOND gives predictions for the redshift in no way inferior to those obtained by WHH for GR. Thus, WHH's claim that GR explains the gravitational redshift measurements better than does T*e*V*e*S/MOND is baseless. None the less, increased precision in the measurement of the gravitational redshift might make it possible in the future to distinguish between GR with NFW haloes and T*e*V*e*S/MOND.

ACKNOWLEDGMENT

We thank M. Milgrom for remarks.

REFERENCES

Angus G. W., Shan H. Y., Zhao H. S., Famaey B., 2007, ApJ, 654, 13
Bekenstein J. D., 2004, Phys. Rev. D, 70, 083509
Carroll S., Divurri V., Trodden M., Turner M. S., 2004, Phys. Rev. D, 70, 043528
Famaey B., Binney J., 2005, MNRAS, 363, 603
Milgrom M., 1983, ApJ, 270, 384
Milgrom M., 2008, New Astron. Rev., 51, 906
Misner C. W., Thorne K. S., Wheeler J. A., 1971, Gravitation. Freeman & Co., San Francisco
Navarro J. F., Frenk C. S., White S. D. M., 1997, ApJ, 490, 493
Pointecouteau E., Silk J., 2005, MNRAS, 364, 654
Sanders R. H., 2003, MNRAS, 342, 901
Sanders R. H., McGaugh S. S., 2002, ARA&A, 40, 263
Skordis C., 2009, Class. Quantum Gravity, 36, 143001
Will C., 1986, Theory and Experiment in Gravitational Physics. Cambridge Univ. Press, Cambridge
Wojtak R., Hansen S. H., Hjorth J., 2011, Nat, 477, 567 (WHH)

This paper has been typeset from a TEX/LATEX file prepared by the author.